The Block Stacking Problem: Obtaining the greatest shift

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Project no: 1126

The aim of my work

- **Research question:** When identical blocks are stacked on top of each other through regular stacking and 90° rotated stacking, what is the greatest shift that can be obtained between the bottom and the top block?

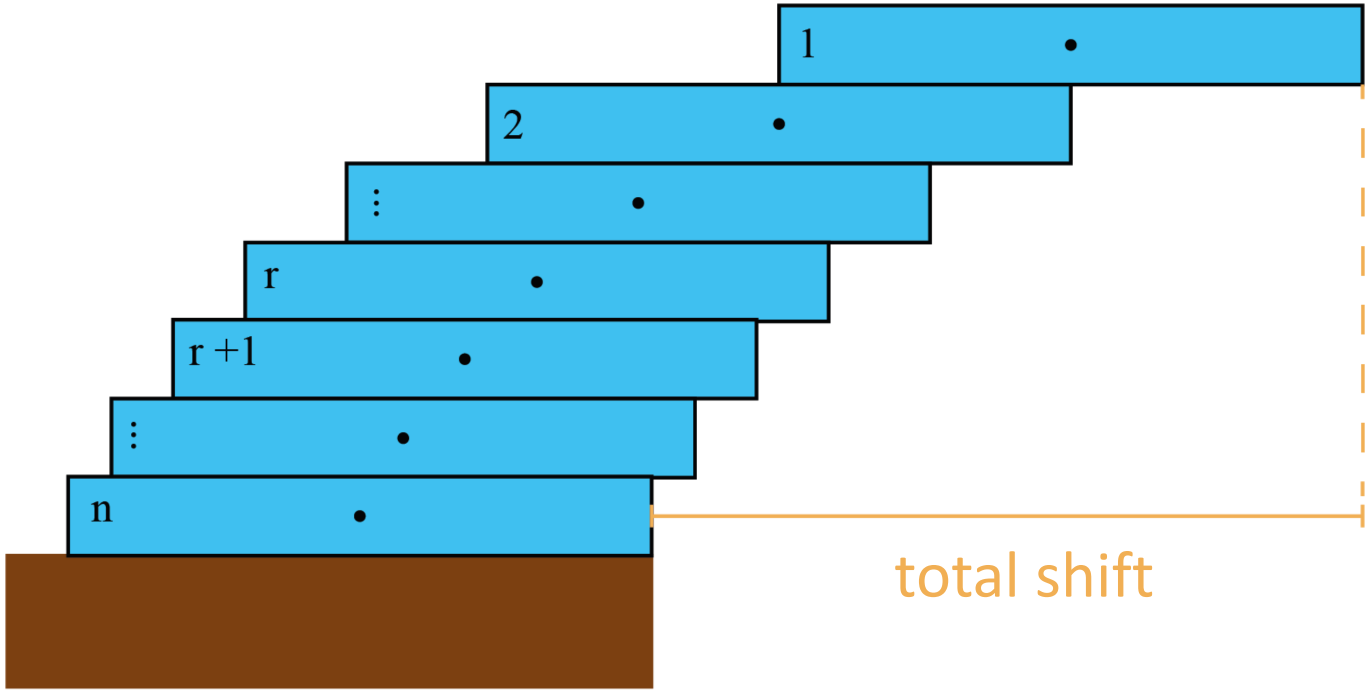

Figure 1. Representation of regular stacking (Original version of the block stacking problem)

The aim of my work

- The alternative version of the block stacking problem that I designed:

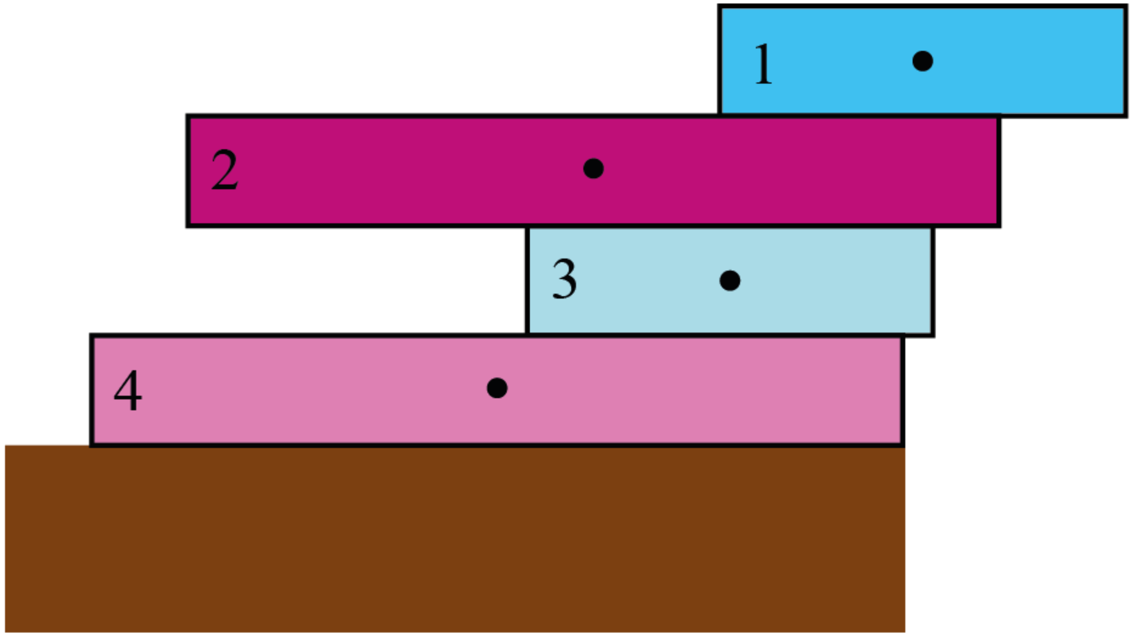

Figure 2. Rotated blocks from the side

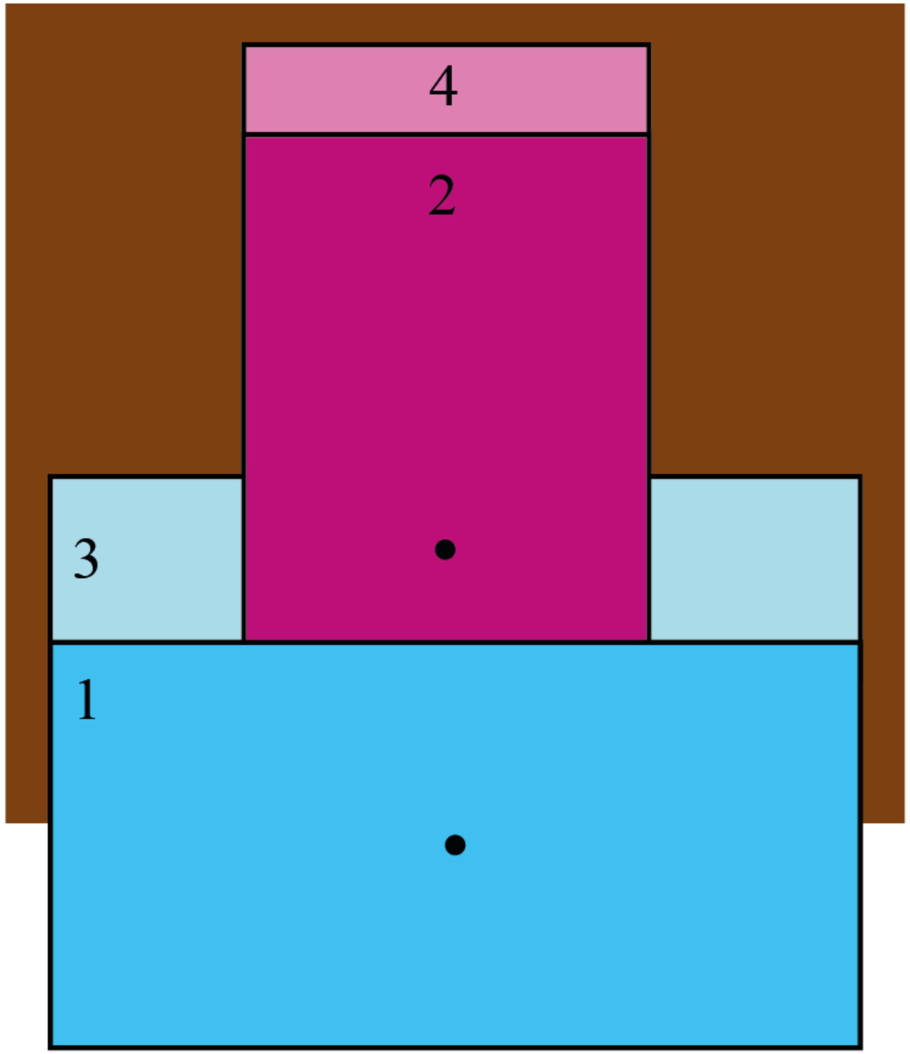

Figure 3. Rotated blocks from the top

My approach to the problem

Regular stacking

- Deriving the centre of gravity (c_n) equation:

$$c_n = \frac{m_a x_a + m_n x_n}{m_a + m_n}$$

- Using proof by induction for the simplification of an expression regarding the shift given by each block
- Extensive algebraic manipulation to solve for the greatest shift in regular stacking
- Proving that the sum of the total shift diverges

$90°$ rotated stacking

- Discovering a trend for the maximum shift by exploring the centre of gravity for specific values of n
- Use of infinite sums
- Use of definite integrals to show that the sum of the total shift diverges
- Using the Maclaurin series expansion for $\ln(1 + x)$ to express the total shift in $90°$ rotated stacking in a comparable form to regular stacking

Main findings and conclusions: Regular stacking

- The greatest shift is obtained through the harmonic series:

$$\sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

- Every following shift is smaller than the previous one.
- As the number of blocks increases without bound, the shift obtained increases without bound as well.

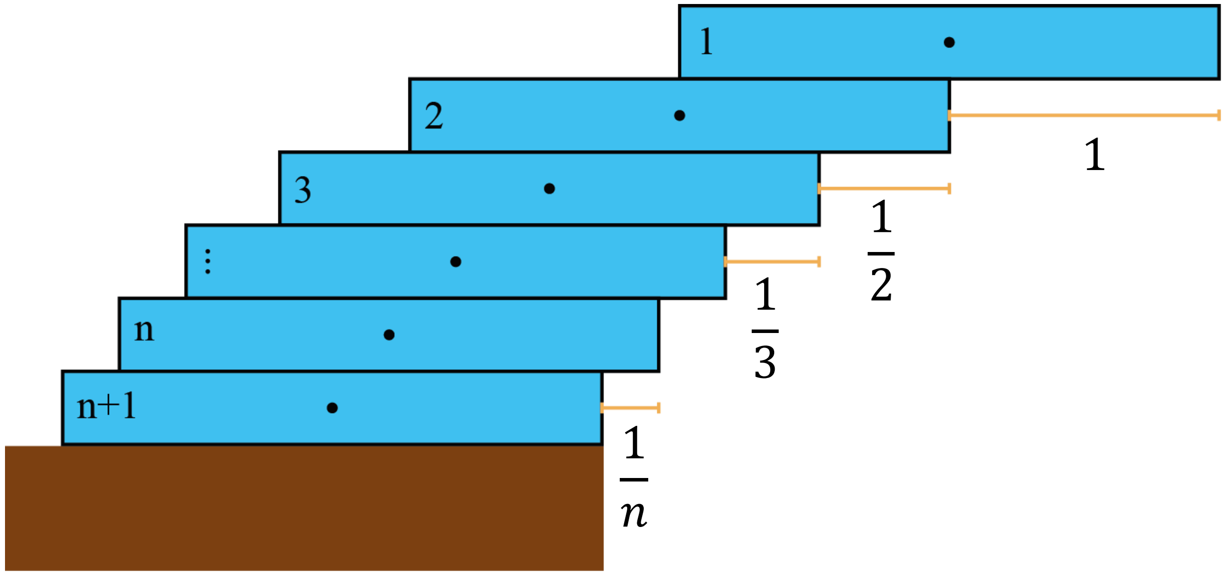

Figure 4. The use of harmonic series to obtain the greatest shift in regular stacking.

Main findings and conclusions: $90°$ rotated stacking

- The total greatest shift is:

$$S_n = \frac{1}{2} \times \sum_{t=1}^{\frac{n}{2}} \frac{1}{2t-1} + \sum_{t=1}^{\frac{n}{2}} \frac{1}{2t}$$

$$S_n \approx \frac{3}{4}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right) - \frac{\ln 2}{4}$$

- Assuming that a very large number of blocks are stacked, the maximum horizontal shift that can be obtained is $\frac{3}{4}$ of the harmonic series.

- As the number of blocks increases without bound, the shift obtained increases without bound as well.
- Stacking the blocks regularly without rotation results in a greater shift than stacking with 90° rotation.