Airplane collision prevention

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Research Question

To what extent can we model possible collisions between two aircrafts in two-dimensions and prevent such collisions from happening?

Interest in the topic

I wanted to investigate automobile traffic, but the mathematical approaches to it seemed daunting once I did some research. Then I considered airplanes, and whether there could be some opportunities for mathematical research in aviation? I found that airplanes have safe zones, which, e.g, could be modeled with circles and their breach could be tracked with segment lines drawn from the center of one circle to another. This was exciting to me and offered the opportunity for making my own mathematical models.

Defining two-dimensional aeroplanes

1. Each plane follows a linear path called the path function, $f_i(x), i = 1{,}2$.
2. Each plane moves at constant velocity and has a velocity function $v_i(t), i = 1{,}2$
3. Each plane has a circular safe zone, defined by some radius value r, around their location, which is some point on the path function.

Path and velocity functions

$f_i(x) = m_i x + c_i$ and $v_i(t) = \overline{v_i}$, where $\overline{v_i}$ is constant.

The mathematics used and important models

I used functional, geometric, and trigonometric approaches in constructing mathematical models for different airplane collision cases.

Categorization of of x & y coordinates based on direction

Direction (quadrant)	I	IV	II	III
Type of slope	$\Delta y, \Delta x$	$-\Delta y, \Delta x$	$\Delta y, -\Delta x$	$-\Delta y, -\Delta x$
y_{ji}	$\overline{v_j}t\,sin\ (\arctan m_j) + y_{j1}$	$-\overline{v_j}t\,sin\ (\arctan -m_j) + y_{j1}$	$\overline{v_j}t\,sin\ (\arctan -m_j) + y_{j1}$	$-\overline{v_j}t\,sin\ (\arctan m_j) + y_{j1}$
x_{ji}	$\overline{v_j}t\,cos\ (\arctan m_j) + x_{j1}$	$\overline{v_j}t\,cos\ (\arctan -m_j) + x_{j1}$	$-\overline{v_j}t\,cos\ (\arctan -m_j) + x_{j1}$	$-\overline{v_j}t\,cos\ (\arctan m_j) + x_{j1}$

Conflict inequality (used to determine if a safe-zone violation is going to occur)

$$(y_{2i} - y_{1i})^2 + (x_{2i} - x_{1i})^2 - (r_1 + r_2)^2 \geq 0$$

Results

The conflict inequality gave accurate results in multiple cases. There came some challenges in trying to “rectify” a conflict situation, i.e, attempting to change a plane’s parameters for collision prevention. But a case like the one on the right was doable with my models.

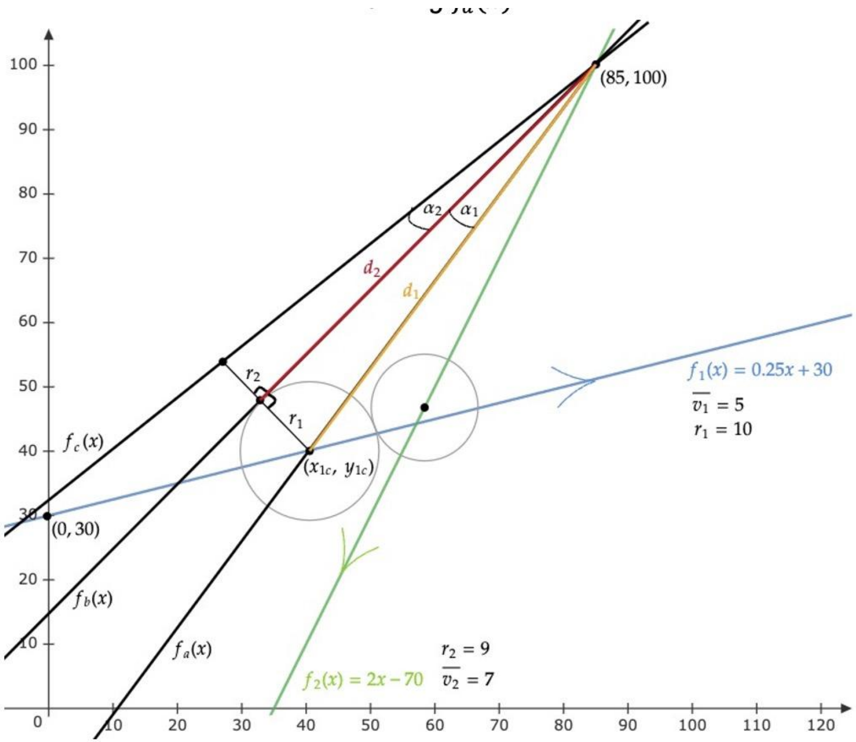

Figure 14: fully labeled Category A pairwise conflict graph.

Conclusion

We can prevent collisions from happening by firstly predicting the net separation of planes and secondly assessing whether a maneuver at the same altitude or a vertical maneuver is appropriate through the angular change that results from changing a plane’s path function.